

Application of VQ to Optimal Transform Coding

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Abstract-In this paper, an optimal adaptive transform coding system based on VQ is proposed. In the proposed approach, the K -means clustering is used to find representative image blocks through which VQ-based masks are found. The set of VQ-based masks is then applied in the selection of transform coefficients. It is shown that the set of VQ-based masks is optimal in the sense of minimum average energy loss. Next, the set of optimal VQ-based masks is applied in JPEG. The modified JPEG with optimal VQ-based masks is termed as OJPEG. Simulation results are provided to justify the optimality in VQ-based masks. Besides, the proposed OJPEG is compared with JPEG in terms of PSNR and bit rate. It indicates that the proposed OJPEG is able to trade little PSNR with significant bit rate reduction when compared with JPEG.

Keywords: VQ, Optimality, Transform Coding

1. Introduction

The main objective in image coding is to reduce the required memory while the reconstructed image is of acceptable quality. Some applications of image coding are communications, multimedia system, and so forth. Among image coding techniques, vector quantization (VQ) [1] and transform coding [2] are two popular approaches. The operation of VQ is quite simple. The first step is to find a codebook or representative patterns from a given set of training patterns. Then each input pattern is assigned to a class index according a similarity measure, which is usually Euclidean distance. The reconstructed pattern is formed from its representative pattern. As for transform coding (TC), it transforms image blocks, from spatial domain to frequency domain, i.e., the information of image block is converted in its corresponding transform coefficients. Note that the quality of reconstructed image is related to the total energy of transform coefficients discarded in the coding process. Consequently, effectively selecting significant transform coefficients implies better quality of reconstructed image.

In this paper, an approach to incorporate VQ into TC is proposed. The application of VQ to TC is not new. Several types of application have been reported. In [3], a DCT-transformed image, based on

the energy criteria, is classified as one of four classes: H, V, D, and L. Then the DCT transform coefficients are partitioned as several sub-vectors for each class. For each sub-vector in a given class, VQ is performed with different codebooks. In [4], VQ is used to construct 'transform book' which is a set of block transforms. By a similarity measure, an input image block is classified and a block transform is adaptively chosen to transform the given image block. In [5], an average optimal vector transform is designed. With the optimal vector transform, VQ is applied in the transform domain of image blocks. In [6], a self-organizing mapping neural network [7], which is considered as neural VQ, is employed in DCT domain because of the topology preserving property. In [8], VQ is applied to classify the linear prediction coefficients of image blocks while VQ is given to classify image blocks and the class index is then mapped to a pre-computed compressed bit stream in [9].

In this paper, an optimal VQ-based transform coding approach is proposed where VQ is used to find optimal masks in the selection of transform coefficients. This paper is organized as follows. First, the K -man clustering is briefly reviewed. Then VQ-based masks are introduced which is followed by the derivation of the optimality in VQ-based masks. Next, the way to apply optimal VQ-based masks to JPEG is given. The modified JPEG with optimal VQ-based masks is called OJPEG. Then simulations are provided to justify the optimal set of VQ-based masks and to compare OJPEG with JPEG in terms of PSNR and bit rate. Finally, conclusions are made.

2. The K -Means Clustering

This section gives a brief review of conventional VQ, the K -means clustering (KMC) [1]. Given a set of image blocks $\{\mathbf{b}_i, \text{ for } 1 \leq i \leq N_b\}$ where N_b is the total number of image blocks, the process of KMC to obtain K representative image blocks is summarized as follows.

- Step 1. Initialize representative image blocks \mathbf{r}_k , for $1 \leq k \leq K$, with random numbers.
- Step 2. For a given \mathbf{b}_i , the similarity measure, Euclidean distance, d_k is calculated as

$$d_k = \|\mathbf{b}_i - \mathbf{r}_k\| \quad (1)$$

for $1 \leq k \leq K$. Then classify \mathbf{b}_i as class j if $d_j = \min_k d_k$.

Step 3. Update representative image block \mathbf{r}_k , for $1 \leq k \leq K$, as

$$\mathbf{r}_k = \frac{1}{N_k} \sum_{i=1}^{N_k} \mathbf{b}_{k,i} \quad (2)$$

where $\mathbf{b}_{k,i}$ is a \mathbf{b}_i belonging to class k and N_k is the total number of image blocks in class k .

Step 4. Stop if the learning process converges. Otherwise, go to Step 2.

After learning, the representative image blocks \mathbf{r}_k , for $1 \leq k \leq K$, obtained from KMC are used to find masks and to classify image blocks in the proposed OJPEG.

3. Optimal VQ-Based Masks

In this section, the way to find VQ-based masks is described in the following. Assume the original image \mathbf{O} is of size $L \times L$ and is partitioned into $\tau \times \tau$ image blocks denoted as $\{\mathbf{b}_i, \text{for } 1 \leq i \leq N_b\}$, where $N_b = (L/\tau)^2$. The way to find VQ-based masks is described in the following.

Step 1. Obtain representative image blocks \mathbf{r}_k by KMC, for $1 \leq k \leq K$.

Step 2. Classify image blocks $\{\mathbf{b}_i, \text{for } 1 \leq i \leq N_b\}$ into K classes. The classified set is denoted as $\{\mathbf{b}_{k,i}, \text{for } 1 \leq k \leq K, 1 \leq i \leq N_k\}$ where N_k is the number of image blocks assigned to class k .

Step 3. Find the transformed image block of $\mathbf{b}_{k,i}$ as $\mathbf{B}_{k,i} = \text{DCT}\{\mathbf{b}_{k,i}\}$ where $\text{DCT}\{\cdot\}$ is the discrete cosine transform [2].

Step 4. Calculate the average energy image block of $\mathbf{B}_{k,i}$ as

$$\bar{\mathbf{B}}_k = \frac{1}{N_k} \sum_{i=1}^{N_k} \mathbf{B}_{k,i} \cdot * \mathbf{B}_{k,i} \quad (3)$$

where operation $\cdot *$ is the element-to-element multiplication.

Step 5. Sort elements of $\bar{\mathbf{B}}_k$ in the descending order by energy.

Step 6. Save the indices of sorted elements in $\bar{\mathbf{B}}_k$ as set \mathbf{S}_k .

Step 7. Through \mathbf{S}_k , find VQ-based masks $\mathbf{A}_{k,M}$ where the subscript M denotes the number of elements selected in \mathbf{S}_k . When parameter M is specified, the first M indices in \mathbf{S}_k are selected to find $\mathbf{A}_{k,M}$.

Masks $\mathbf{A}_{k,M}$ are matrices with elements of zero or one. If the indices in \mathbf{S}_k are selected, the corresponding elements in $\bar{\mathbf{B}}_k$ are set to one and set to zero, otherwise. The resulted matrix is the mask for class k . The set of masks $\mathbf{A}_{k,M}$ will be applied in the selection of transform coefficients. Since masks $\mathbf{A}_{k,M}$ are fixed for all classes, thus the number of selected coefficients is same for all $\mathbf{B}_{k,i}$.

4. Optimality in VQ-Based Masks

In this section, VQ-based masks obtained in the previous section are shown to be optimal in the sense of minimum average loss. Note that \mathbf{r}_k is obtained as

$$\mathbf{r}_k = \frac{1}{N_k} \sum_{i=1}^{N_k} \mathbf{b}_{k,i} \quad (4)$$

where $\mathbf{b}_{k,i}$ is the i th image block in class k . By a linear orthogonal transformation T such as DCT, the transformed \mathbf{r}_k is found as

$$\begin{aligned} \mathbf{R}_k &= T\{\mathbf{r}_k\} = T\left\{\frac{1}{N_k} \sum_{i=1}^{N_k} \mathbf{b}_{k,i}\right\} \\ &= \frac{1}{N_k} \sum_{i=1}^{N_k} T\{\mathbf{b}_{k,i}\} = \frac{1}{N_k} \sum_{i=1}^{N_k} \mathbf{B}_{k,i} \end{aligned} \quad (5)$$

where $\mathbf{B}_{k,i} = T\{\mathbf{b}_{k,i}\}$ and elements of \mathbf{R}_k are calculated as

$$R_k(m,l) = \frac{1}{N_k} \sum_{i=1}^{N_k} B_{k,i}(m,l) \quad (6)$$

In (6), $B_{k,i}(m,l)$ are elements of $\mathbf{B}_{k,i}$. By squaring both sides of (6), we have

$$\begin{aligned} R_k^2(m,l) &= \left[\frac{1}{N_k} \sum_{i=1}^{N_k} B_{k,i}(m,l)\right]^2 \\ &= \frac{1}{N_k} \left[\frac{1}{N_k} \sum_{i=1}^{N_k} B_{k,i}^2(m,l)\right] + C_k(m,l) \\ &= \frac{1}{N_k} \bar{B}_k(m,l) + C_k(m,l) \end{aligned} \quad (7)$$

where

$$\bar{B}_k(m,l) = \frac{1}{N_k} \sum_{i=1}^{N_k} B_{k,i}^2(m,l) \quad (8)$$

is the average energy of the (m,l) element in image blocks $\mathbf{B}_{k,i}$ and $C_k(m,l)$ denotes the sum of cross terms in expanding $\left[\frac{1}{N_k} \sum_{i=1}^{N_k} B_{k,i}(m,l)\right]^2$. Note that

the element-to-element relationship between $R_k^2(m,l)$ and $\bar{B}_k(m,l)$ has been established.

Now, define

$$\tilde{R}_k^2(m,l) = N_k [R_k^2(m,l) - C_k(m,l)] = \bar{B}_k(m,l) \quad (9)$$

With elements defined in (9), a new block $\tilde{\mathbf{R}}_k$ is formed. Note that $\tilde{\mathbf{R}}_k$ is identical to $\bar{\mathbf{R}}_k$ in (3). As described in the previous section, the set \mathbf{S}_k is found from $\tilde{\mathbf{R}}_k$. Given parameter M , mask $\mathbf{A}_{k,M}$ is obtained. Denote $\hat{\mathbf{R}}_{k,M}$ as $\tilde{\mathbf{R}}_k$ with M elements selected by $\mathbf{A}_{k,M}$, i.e., $\hat{\mathbf{R}}_{k,M} = \mathbf{A}_{k,M} * \tilde{\mathbf{R}}_k$. Then the average energy loss in class k is given as

$$E_{k,M} = \sum_{m=1}^{\tau} \sum_{l=1}^{\tau} [\tilde{\mathbf{R}}_k^2(m,l) - \hat{\mathbf{R}}_{k,M}^2(m,l)] \quad (10)$$

To see $E_{k,M}$ is minimum, the energy-invariant property in an orthogonal transformation is applied [2]. For image blocks in class k , it implies that

$$\begin{aligned} & \frac{1}{N_k} \sum_{i=1}^{N_k} \sum_{m=1}^{\tau} \sum_{l=1}^{\tau} b_{k,i}^2(m,l) \\ &= \frac{1}{N_k} \sum_{i=1}^{N_k} \sum_{m=1}^{\tau} \sum_{l=1}^{\tau} B_{k,i}^2(m,l) \quad (11) \\ &= \sum_{m=1}^{\tau} \sum_{l=1}^{\tau} \bar{B}_k(m,l) \end{aligned}$$

where (8) is applied in the second equality. With (10) and (11), it is clear that $E_{k,M}$ is of minimum average energy loss since $\hat{\mathbf{R}}_{k,M}$ retains M most significant $\bar{B}_k(m,l)$. Consequently, $E_{k,M} \leq E'_{k,M}$ for all M , where $E'_{k,M}$ is an average energy loss with arbitrary mask $\mathbf{A}'_{k,M}$. When $\mathbf{A}'_{k,M} = \mathbf{A}_{k,M}$, the equality $E_{k,M} = E'_{k,M}$ holds. This implies that $\mathbf{A}_{k,M}$ is an optimal mask for class k in the sense of minimum average energy loss.

Next, consider the average energy loss in the original image \mathbf{O} . With masks $\mathbf{A}_{k,M}$ for $1 \leq k \leq K$, the overall average energy loss for image \mathbf{O} , E_M , is given as

$$E_M = \frac{1}{K} \sum_{k=1}^K E_{k,M} \quad (12)$$

The overall average energy loss E_M is minimum because of the optimality principle of dynamic programming [11]. Thus $E_M \leq E'_M$ for all M , where E'_M is an overall average energy loss with arbitrary set of masks $\mathbf{A}'_{k,M}$ for $1 \leq k \leq K$. When $\mathbf{A}'_{k,M} = \mathbf{A}_{k,M}$ for all k , the equality $E_M = E'_M$ holds. This concludes that $\mathbf{A}_{k,M}$, for $1 \leq k \leq K$, form a set of optimal masks for image \mathbf{O} in the sense of minimum average energy loss.

5. JPEG with Optimal VQ-Based Masks

In this section, optimal VQ-based masks $\mathbf{A}_{k,M}$ are applied to JPEG [10]. The coding process of JPEG is shown in Figure 1 where ZZS and DPCM stand for zigzag scan and differential pulse code modulation, respectively. Here, JPEG is modified to incorporate optimal masks $\mathbf{A}_{k,M}$ which are used in the selection of transform coefficients. The modified JPEG is termed as optimal JPEG (OJPEG) and is described in the following.

Given an $L \times L$ image \mathbf{O} , the proposed OJPEG with a specified M is described in the following.

- Step 1. Partition image \mathbf{O} as $\tau \times \tau$ image blocks $\{\mathbf{b}_i, \text{ for } 1 \leq i \leq N_b\}$ where $\tau < L$ and $N_b = (L/\tau)^2$.
- Step 2. Classify image blocks $\{\mathbf{b}_i, \text{ for } 1 \leq i \leq N_b\}$ into K classes by KMC. The classified image block \mathbf{b}_i is denoted as $\mathbf{b}_{k,i}$ when it belongs to class k .
- Step 3. Find set \mathbf{S}_k and then optimal masks $\mathbf{A}_{k,M}$, for $1 \leq k \leq K$.
- Step 4. Obtain $\hat{\mathbf{B}}_{k,i} = \mathbf{A}_{k,M} * \mathbf{b}_{k,i}$.
- Step 5. Quantize and code $\hat{\mathbf{B}}_{k,i}$ as in JPEG except the order of $\mathbf{S}_{k,M}$ is used rather than ZZS where $\mathbf{S}_{k,M}$ is the set with the first M elements in \mathbf{S}_k .
- Step 6. Decode $\hat{\mathbf{B}}_{k,i}$ through $\mathbf{S}_{k,M}$ and find the reconstructed image block $\hat{\mathbf{b}}_{k,i} = IDCT\{\hat{\mathbf{B}}_{k,i}\}$ where $IDCT\{\cdot\}$ is the inverse DCT.
- Step 7. Obtain the reconstructed image of $\hat{\mathbf{O}}$ from $\hat{\mathbf{b}}_{k,i}$.
- Step 8. Calculate the peak signal-to-noise (PSNR) of $\hat{\mathbf{O}}$ as

$$PSNR = 10 \log \frac{255^2}{MSE} \quad (16)$$

where

$$MSE = \frac{1}{L^2} \sum_{i=1}^L \sum_{j=1}^L [O(i,j) - \hat{O}(i,j)]^2 \quad (17)$$

and $O(i,j)$, $\hat{O}(i,j)$ are elements of \mathbf{O} , $\hat{\mathbf{O}}$, respectively.

- Step 9. Calculate the bit rate (bit/pixel), BR, for reconstructed image $\hat{\mathbf{O}}$ as

$$BR = \frac{B_u \times 8}{L \times L} \quad (18)$$

where B_u denotes the total number of bytes used in the bit stream obtained in Step 5 and bytes used to indicate classes and to

store $\mathcal{S}_{k,M}$.

Note that in OJPEG the overhead, $(L/\tau)^2 \times \lceil \log_2 K \rceil + M \times \lceil \log_2(\tau \times \tau) \rceil$ bits, is required to indicate class indices and masks $\mathcal{A}_{k,M}$.

6. Simulation Results

This section is divided into two subsections. First, the optimality of VQ-based masks is verified. Then JPEG and OJPEG are compared in terms of PSNR and BR. To distinguish the results, no quantization and coding are applied in the optimality test. Moreover, all original image size is 512×512 and image block size is 8×8 . That is, $L = 512$ and $\tau = 8$. Four images Baboon, Lena, Harbor, and Peppers are used in the simulation.

6.1 Optimality verification

In this subsection, image Lena is used as an example to justify the optimality of VQ-based masks. In the example, parameters K and M are specified as 4 and 16, respectively. Representative image blocks r_k are obtained by KMC. With $\mathcal{B}_{k,i}$, $\bar{\mathcal{B}}_k$ is obtained as in (3), for $1 \leq k \leq 4$. Then masks $\mathcal{A}_{k,M}$ are found by $\bar{\mathcal{B}}_k$ and \mathcal{S}_k . For the convenience of presentation, the two-dimensional index in $\mathcal{A}_{k,M}$ is mapped to one-dimensional index. The conversion is given in Figure 2. With the new index notation, the 16 selected elements with value one in $\mathcal{A}_{k,M}$ are given in Table 1. Note that masks $\mathcal{A}_{k,M}$ in Table 1 are ordered sets whose order are significant.

Note that the average energy loss is inverse proportional to PSNR in reconstructed image, i.e., low average energy loss means high PSNR and vice versa. Thus the reconstructed image with $\mathcal{A}_{k,M}$ has highest PSNR which is 36.139 dB in the example. In other words, PSNR with masks $\mathcal{A}'_{k,M}$ is less than 36.139 dB if $\mathcal{A}'_{k,M} \neq \mathcal{A}_{k,M}$ for some k . If this is true, the optimality of $\mathcal{A}_{k,M}$ is verified. To test the optimality of $\mathcal{A}_{k,M}$, an element in Table 1 is randomly chosen and replaced with other element not in $\mathcal{A}_{k,M}$. Twelve experiments have been performed and their results are summarized in Table 2. As expected, PSNR in Table 2 are all less than 36.139 dB which is obtained by masks $\mathcal{A}_{k,M}$. Therefore, the optimality is verified.

6.2 Comparison between JPEG and OJPEG

In this subsection, the comparison of JPEG and OJPEG is made in terms of PSNR and BR. By Figure 1, the PSNR obtained from JPEG are 28.1197 dB,

36.4003 dB, 34.7112 dB, and 30.5387 dB, for images Baboon, Lena, Peppers, and Harbor, respectively. The BR for images Baboon, Lena, Peppers, and Harbor, are 1.6479, 1.0641, 1.0996, and 1.2796, respectively. The comparison results on PSNR for JPEG and OJPEG, up to $\Delta PSNR$ around 0.1 dB, are given in Table 3 where $\Delta PSNR = PSNR_{JPEG} - PSNR_{OJPEG,M}$. Notations $PSNR_{JPEG}$ and $PSNR_{OJPEG,M}$ stand for the PSNR for JPEG and OJPEG with parameter M , respectively. In the case of $M = 28$ for image Lena, $\Delta PSNR = 36.4003 - 36.3334 = 0.0669$ which means OJPEG is worse than JPEG by 0.0669 dB in PSNR. The comparison results on BR, corresponding up to $\Delta PSNR$ around 0.1 dB, are also shown in Table 3 where $\Delta BR = BR_{JPEG} - BR_{OJPEG,M}$ and BR_{JPEG} , $BR_{OJPEG,M}$ denote BR resulted from JPEG and OJPEG with parameter M , respectively. In the calculation of $BR_{OJPEG,M}$, the overhead, $(L/\tau)^2 \times \lceil \log_2 K \rceil + M \times \lceil \log_2(\tau \times \tau) \rceil$ bits, has been put into account. In the case of $M = 28$ for image Lena, $\Delta BR = 1.0641 - 0.7289 = 0.3352$. This suggests that JPEG takes 0.3352 BR more than OJPEG. By Table 3, it indicates that JPEG pays ΔBR bit rate to have $\Delta PSNR$ dB improvement on PSNR when compared with OJPEG. From the other viewpoint, it can be said that OJPEG trades $\Delta PSNR$ dB in PSNR for ΔBR bit rate reduction. Take an example. For image Lena, JPEG uses 0.3352 BR to obtain 0.0669 dB improvement on PSNR when compared with the case in OJPEG of $M = 28$. Or it can be said that OJPEG pays 0.0669 dB in PSNR to exchange 0.3352 bit rate reduction. Even in the extreme cases in Table 3, images Baboon, Lena, Peppers, and Harbor still have bit rate reduction 0.1072 ($M = 48$), 0.3352 ($M = 28$), 0.3014 ($M = 32$), and 0.0971 ($M = 48$), at the cost of 0.1305 dB, 0.0669 dB, 0.1020 dB, and 0.0981 dB degradation in PSNR, respectively. If higher PSNR degradation is acceptable, i.e., smaller M , BR can be reduced further in OJPEG. By our experiences, $M = 28$ is sufficient for most of cases. As shown in Table 3, in this case the bit rate reduction for images Baboon, Lena, Peppers, and Harbor are 0.3905, 0.3352, 0.3308, and 0.3340, respectively. In summary, the simulation results indicate that OJPEG is able to effectively trade little PSNR with significant bit rate reduction when compared with JPEG.

7. Conclusions

In this paper, an approach to find masks based on VQ is proposed. The set of VQ-based masks is proven optimal in the sense of minimum average energy loss. Then optimal VQ-based masks are then applied in transform coding to select of significant transform coefficients. Next, a coding system

modified from JPEG is proposed which is termed as OJPEG. Basically, OJPEG consists of three stages. In the first stage, representative image blocks are found by the K -means clustering. Then for each class reorder the transform coefficients by their average energies and save the reordered indices of transform coefficients as set S_k for $1 \leq k \leq K$ where K is the number of classes used. By S_k , optimal VQ-based masks $A_{k,M}$ are obtained. At the second stage, the set of optimal masks $A_{k,M}$ is applied to select M significant transform coefficients. In the final stage, the M selected transform coefficients are quantized and coded as in JPEG where the scan order is based on S_k . Simulations are provided to verify the optimality of VQ-based masks. The results are consistent with the theoretical results. Besides, OJPEG is compared with JPEG in terms of PSNR and BR. From Table 3, it indicates that Δ PSNR are in a decreasing order and get smaller and smaller, as M increases, for all images. It implies that OJPEG is able to select transform coefficients by their significance order. Though the PSNR of OJPEG is always less than that from JPEG except the case $M = 64$, OJPEG is able to trade little PSNR with significant bit rate reduction since the effectiveness of OJPEG in the selection of significant transform coefficients. Simulation results in Table 3 have confirmed the tradeoff between PSNR and BR for OJPEG in comparison with JPEG. To sum up, through simulation results the optimality in VQ-based masks is verified, the application of optimal VQ-based masks to JPEG is shown, and the tradeoff between PSNR and BR for JPEG and OJPEG is evaluated.

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References

- [1] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*, Kluwer Academic Publishers, 1992.
- [2] K. R. Rao and J. J. Hwang, *Techniques and Standards for Image, Video, and Audio Coding*, Prentice Hall, 1996.
- [3] J. Y. Nam and K. R. Rao, "Image Coding Using a Classified DCT/VQ Based on Two-Channel Conjugate Vector Quantization," *IEEE Trans. on Circuits and Systems for Video Technology*, Vol. 1, No. 4, pp. 325-336, 1991.
- [4] H. Caglar, S. Güntürk, B. Sankur, and E. Anarim, "VQ-Adaptive Block Transform Coding of Images," *IEEE Trans. on Image Processing*, Vol. 7, No. 1, pp. 110-115, 1998.
- [5] G. Sudhir, M. L. Liou, and John C. M. Lee, "Average Optimal Vector Transform for VQ-Based Image and Video Compression," *IEEE Trans. on Circuits and Systems for Video Technology*, Vol. 9, No. 4, pp. 617-629, 1999.
- [6] C. Amerijckx, M. Verleysen, and P. Thissen, and J.-D. Legat, "Image Compression by Self-Organized Kohonen Map," *IEEE Trans. on Neural Networks*, Vol. 9, No. 3, 1998.
- [7] S. Haykin, *Neural Networks – A Comprehensive Foundation*, Macmillan, 1994.
- [8] H. Jafarkhani and N. Farvardin, "Adaptive Image Coding Using Spectral Classification," *IEEE Trans. on Image Processing*, Vol. 7, No. 4, April, 1998.
- [9] R. L. de Queiroz and P. Fleckenstein, "Very Fast JPEG Compression Using Hierarchical Vector Quantization," *IEEE Signal Processing Letters*, Vol. 7, No. 5, May 2000.
- [10] W. B. Pennebaker and J. L. Mitchell, *JPEG Still Image Data Compression Standard*, New York, NY: Van Nostrand Reinhold, 1993.
- [11] A. J. Viterbi and J. K. Omura, "Trellis Encoding of Memoryless Discrete-Time Sources with a Fidelity Criterion," *IEEE Trans. on Information Theory*, pp. 325-332, May, 1974.

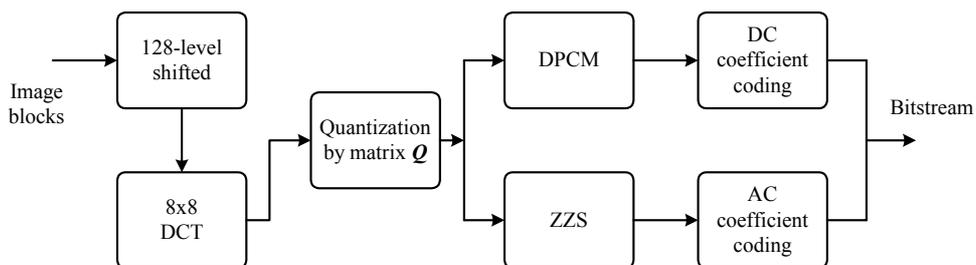


Figure 1. The coding process of JPEG

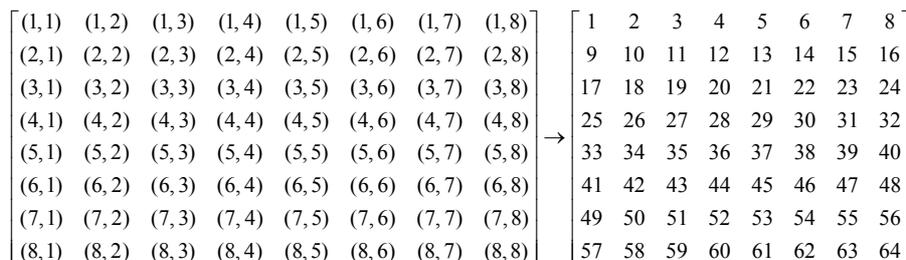


Figure 2. Index conversion for $A_{k,M}$

Table 1. The selected elements in $A_{k,M}$

Ranking index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$A_{1,16}$	1	2	9	10	3	18	17	11	4	19	12	26	27	20	5	25
$A_{2,16}$	1	2	9	3	10	11	4	18	17	19	12	5	20	25	26	27
$A_{3,16}$	1	2	9	10	3	11	17	18	19	4	12	20	5	25	27	26
$A_{4,16}$	1	2	9	3	10	4	11	17	18	19	12	5	20	6	13	26

Table 2. Tests on the optimality of $A_{k,M}$

Mask changed	$A_{1,16}$	$A_{1,16}$	$A_{1,16}$	$A_{2,16}$	$A_{2,16}$	$A_{2,16}$	$A_{3,16}$	$A_{3,16}$	$A_{3,16}$	$A_{4,16}$	$A_{4,16}$	$A_{4,16}$
Original element	25	5	4	20	19	26	27	12	19	6	17	13
Substitute element	33	13	6	28	21	34	36	13	28	14	25	21
PSNR	36.116	36.130	36.057	36.027	35.759	36.006	36.076	36.023	35.946	36.085	35.886	36.093

Table 3. Comparison results for JPEG and OJPEG

Value of M	Baboon		Lena		Peppers		Harbor	
	$\Delta PSNR$	ΔBR						
16	3.7768	0.7027	1.4043	0.5051	1.4948	0.4967	3.2644	0.5926
20	2.8741	0.5675	0.6532	0.4250	0.7699	0.4277	2.5904	0.4729
24	2.1002	0.4591	0.3439	0.3781	0.3863	0.3707	1.8632	0.3939
28	1.5032	0.3905	0.0669	0.3352	0.1660	0.3308	1.2707	0.3340
32	0.9240	0.3014	—	—	0.1020	0.3014	0.8573	0.2818
36	0.5734	0.2472	—	—	—	—	0.5459	0.2093
40	0.3406	0.1977	—	—	—	—	0.3247	0.1656
44	0.1890	0.1639	—	—	—	—	0.1743	0.1313
48	0.1305	0.1072	—	—	—	—	0.0981	0.0971